10.2 Simplifying Radicals

Objectives: Be able to simplify radicals involving products and quotients.

Radical Expression: an expression that contains a radical (aka Square root)

You can simplify a radical expression if:

1. The **radicand** has no **perfect square factors** other than 1.

   | 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144,... |
   | number under the √ |
Perfect Squares:

<table>
<thead>
<tr>
<th>$2^2$</th>
<th>4</th>
<th>$7^2$</th>
<th>49</th>
<th>$12^2$</th>
<th>144</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^2$</td>
<td>9</td>
<td>$8^2$</td>
<td>64</td>
<td>$13^2$</td>
<td>169</td>
</tr>
<tr>
<td>$4^2$</td>
<td>16</td>
<td>$9^2$</td>
<td>81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5^2$</td>
<td>25</td>
<td>$10^2$</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6^2$</td>
<td>36</td>
<td>$11^2$</td>
<td>121</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*These are the factors you are looking for.*
Simplify:

\[ \sqrt{196} \quad 14 \]

\[ \sqrt{160} \quad \frac{\sqrt{16 \cdot 10}}{\sqrt{16} \cdot \sqrt{10}} \]

\[ 4 \sqrt{10} \]
Simplify:

\[\sqrt{72}\]
\[= \frac{\sqrt{36 \cdot 2}}{\sqrt{36 \cdot \sqrt{2}}}\]
\[= \frac{6\sqrt{2}}{\sqrt{2}}\]

\[\sqrt{125}\]
\[= \frac{\sqrt{25 \cdot 5}}{\sqrt{25 \cdot \sqrt{5}}}\]
\[= \frac{5\sqrt{5}}{5\sqrt{5}}\]
Simplify:

\[ \sqrt{112} \]

\[ \frac{\sqrt{16} \cdot 7}{\sqrt{16} \cdot \sqrt{7}} \]

\[ \frac{4 \sqrt{7}}{} \]

\[ \sqrt{98} \]

\[ \frac{\sqrt{49} \cdot 2}{\sqrt{49} \cdot \sqrt{2}} \]

\[ \frac{7 \sqrt{2}}{} \]
Simplifying Square Roots

Concept: Simplifying square roots

Remember: A simplified square root radical contains no square factors other than 1.

Example: Simplify: \( \sqrt{125x^4} \)

Look for square factors.

\[
\sqrt{125x^4} = \sqrt{25x^4 \cdot 5}
\]

\[
= \sqrt{25x^4} \cdot \sqrt{5}
\]

Product property of square roots.

\[
= 5|x^2|\sqrt{5}
\]

25 = 5 and \( x^4 = (x^2)^2 \).

Write the greatest square factor of the radicand. Then simplify. Use a calculator to check when possible.

1. \( \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \)

2. \( \sqrt{300} = \sqrt{100 \cdot 3} = 10\sqrt{3} \)

3. \( \sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3} \)

4. \( \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2} \)

Simplify. Use a calculator to check when possible.

5. \( \sqrt{80} \approx 8.94 \)

6. \( \sqrt{60} \approx 7.75 \)

7. \( \sqrt{72} \approx 8.49 \)

8. \( \sqrt{500} \approx 22.36 \)

9. \( \sqrt{600} \approx 24.49 \)

10. \( \sqrt{120} \approx 10.95 \)

Concept: Simplifying square roots containing variable expressions

Remember: The radicand of a square radical must be equal to or greater than 0.

Example: For what value of \( x \) will \( \sqrt{-3x + 15} \) be a real number?

\( -3x + 15 \geq 0 \)

The radicand must be nonnegative.

\( -3x \geq -15 \)

Solve the inequality.

\( x \leq 5 \)

Reverse the direction of the inequality.

For all \( x \leq 5 \), \( \sqrt{-3x + 15} \) is a real number.

Find the values of \( x \) that make each radical expression a real number.

11. \( \sqrt{x - 8} \)

12. \( \sqrt{-2x - 10} \)

13. \( \sqrt{-3x + 1} \)

14. \( \sqrt{x + 12} \)

15. \( \sqrt{2x - 1} \)

16. \( \sqrt{3x + 2} \)

Evaluate for the given value of the variable. Then simplify, if possible.

17. \( \sqrt{4y + 1}, y = 2 \) \( \sqrt{9} = 3 \)

18. \( \sqrt{3x - 1}, x = 7 \)

19. \( \sqrt{-4b + 7}, b = -2 \)

20. \( \sqrt{-2n + 16}, n = 6 \)

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