Examples for each lesson:

Lesson 11.1

Polygons

A **polygon** is a closed plane figure formed by three or more line segments that meet at points called vertices. You can classify a polygon by the number of sides and the number of angles that it has.

**Congruent** figures have the same size and shape. In a regular polygon, all sides are congruent and all angles are congruent.

Classify the polygon below.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Sides</th>
<th>Angles</th>
<th>Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Nonagon</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Decagon</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

How many sides does this polygon have? **5 sides**

How many angles does this polygon have? **5 angles**

Name the polygon. **Pentagon**

Are all the sides congruent? **No**

Are all the angles congruent? **No**

So, the polygon above is a pentagon. It is **not** a regular polygon.

More information on this strategy is available on Animated Math Model #40.
Lesson 11.2

Triangles

You can classify triangles by the length of their sides and by the measure of their angles. **Classify each triangle.**

Use a ruler to measure the side lengths.

- **equilateral triangle**
  All sides are the same length.

- **isosceles triangle**
  Two sides are the same length.

- **scalene triangle**
  All sides are different lengths.

Use the corner of a sheet of paper to classify the angles.

- **acute triangle**
  All three angles are acute.

- **obtuse triangle**
  One angle is obtuse. The other two angles are acute.

- **right triangle**
  One angle is right. The other two angles are acute.

Classify the triangle according to its side lengths.
It has two congruent sides.
**The triangle is an isosceles triangle.**

Classify the triangle according to its angle measures.
It has one right angle.
**The triangle is a right triangle.**

More information on this strategy is available on Animated Math Model #41.
Lesson 11.3

Quadrilaterals

You can use this chart to help you classify quadrilaterals.

- **Parallelogram**
  - Quadrilateral
  - Opposite sides are parallel
  - Opposite sides are congruent

- **Trapezoid**
  - Quadrilateral
  - Exactly one pair of parallel sides

- **Rectangle**
  - Parallelogram
  - 4 right angles
  - 2 pairs of perpendicular sides

- **Rhombus**
  - Parallelogram
  - 4 congruent sides

- **Square**
  - Rhombus
  - Rectangle

Classify the figure.

The figure has 4 sides, so it is a quadrilateral. The figure has exactly one pair of parallel sides, so it is a trapezoid.

Quadrilateral, trapezoid

More information on this strategy is available on Animated Math Model #42.

Lesson 11.4

Problem Solving • Properties of Two-Dimensional Figures

Haley thinks hexagon ABCDEF has 6 congruent sides, but she does not have a ruler to measure the sides. Are the 6 sides congruent?

<table>
<thead>
<tr>
<th>Read the Problem</th>
<th>Solve the Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What do I need to find?</strong></td>
<td>Trace the hexagon and cut out the shape.</td>
</tr>
<tr>
<td>I need to determine if sides $AB$, $BC$, $CD$, $DE$, $EF$, and $FA$ have the same length</td>
<td><strong>Step 1</strong> Fold the hexagon to match the sides $AB$ and $ED$, sides $FE$ and $FA$, and sides $CD$ and $CB$.</td>
</tr>
<tr>
<td><strong>What information do I need to use?</strong></td>
<td>The sides match, so they are congruent.</td>
</tr>
<tr>
<td>The figure is a hexagon with 6 sides and 6 congruent angles.</td>
<td><strong>Step 2</strong> Fold along the diagonal between $B$ and $E$ to match sides $BA$ and $BC$, sides $AF$ and $CD$, and sides $EF$ and $ED$. Fold along the diagonal between $A$ and $D$ to match sides $AF$ and $AB$, sides $FE$ and $BC$, and sides $DE$ and $DC$.</td>
</tr>
<tr>
<td><strong>How will I use the information?</strong></td>
<td><strong>Step 3</strong> Use logic to match sides $AB$ and $CD$, sides $AB$ and $EF$, sides $BC$ and $DE$, and sides $DE$ and $FA$. The sides match, so they are congruent.</td>
</tr>
<tr>
<td>I will act it out by tracing the figure and then folding the figure to match all the sides to see if they are congruent</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 11.5

Three-Dimensional Figures

A **polyhedron** is a solid figure with faces that are polygons. You can identify a polyhedron by the shape of its faces.

A **pyramid** is a polyhedron with one polygon base. The lateral faces of a pyramid are triangles that meet at a common vertex.

- **Triangular pyramid**
  - The base and faces are triangles.

- **Rectangular pyramid**
  - The base is a rectangle.

- **Square pyramid**
  - The base is a square.

- **Pentagonal pyramid**
  - The base is a pentagon.

- **Hexagonal pyramid**
  - The base is a hexagon.

A **prism** is a polyhedron with two congruent polygons as bases. The lateral faces of a prism are rectangles.

- **Triangular prism**
  - The two bases are triangles.

- **Rectangular prism**
  - All faces are rectangles.

- **Square prism or cube**
  - All faces are squares.

- **Pentagonal prism**
  - The two bases are pentagons.

- **Hexagonal prism**
  - The two bases are hexagons.

A solid figure with curved surfaces is **not a polyhedron**.

- **Cone**
  - The one base is a circle.

- **Sphere**
  - There is no base.

- **Cylinder**
  - The two bases are circles.

Classify the solid figure. Write **prism**, **pyramid**, **cone**, **cylinder**, or **sphere**.

The solid figure has one base.
The rest of its faces are triangles.
So, the solid figure is a **pyramid**.

More information on this strategy is available on Animated Math Model #43.
Lesson 11.6

Unit Cubes and Solid Figures

A unit cube is a cube that has a length, width, and height of 1 unit. You can use unit cubes to build a rectangular prism.

Count the number of cubes used to build the rectangular prism.

The length of the prism is made up of ___ unit cubes.
The width of the prism is made up of ___ unit cubes.
The height of the prism is made up of ___ unit cube.
The number of unit cubes used to build the rectangular prism is ___.

Lesson 11.7

Understand Volume

The volume of a rectangular prism is equal to the number of unit cubes that make up the prism. Each unit cube has a volume of 1 cubic unit.

Find the volume of the prism. 1 unit cube = 1 cubic inch

Step 1 Count the number of unit cubes in the bottom layer of the prism.
There are ___ unit cubes that make up the length of the first layer.
There are ___ unit cubes that make up the width of the first layer.
There is ___ unit cube that makes up the height of the first layer.
So, altogether, there are ___ unit cubes that make up the bottom layer of the prism.

Step 2 Count the number of layers of cubes that make up the prism.
The prism is made up of ___ layers of unit cubes.

Step 3 Find the total number of cubes that fill the prism.
Multiply the number of layers by the number of cubes in each layer.

\[3 \times 8 = 24\] unit cubes

Each unit cube has a volume of 1 cubic inch. So, the volume of the prism is 24 \times 1, or 24 cubic inches.
Lesson 11.8

Estimate Volume

You can estimate the volume of a larger box by filling it with smaller boxes.

Mario packs boxes of markers into a large box. The volume of each box of markers is 15 cubic inches. Estimate the volume of the large box.

The volume of one box of markers is \(15\) cubic inches.

Use the box of markers to estimate the volume of the large box.

- The large box holds \(\frac{2}{5}\) layers of boxes of markers, a top layer and a bottom layer. Each layer contains \(\frac{10}{5}\) boxes of markers. So, the large box holds about \(2 \times 10\), or \(\frac{20}{5}\) boxes of markers.

- Multiply the volume of 1 box of markers by the estimated number of boxes of markers that fit in the large box.

\[
\frac{20}{5} \times \frac{15}{5} = 300
\]

So, the volume of the large box is about \(300\) cubic inches.

Lesson 11.9

Volume of Rectangular Prisms

Jorge wants to find the volume of this rectangular prism. He can use cubes that measure 1 centimeter on each side to find the volume.

**Step 1** The base has a length of 2 centimeters and a width of 3 centimeters. Multiply to find the area of the base.

\[
\text{Base} = \frac{2}{5} \times \frac{3}{5} \text{ cm}^2
\]

**Step 2** The height of the prism is 4 centimeters. Add the number of cubes in each layer to find the volume.

**Remember:** Each layer has 6 cubes.

**Step 3** Count the cubes \(24\) cubes

Multiply the base and the height to check your answer.

\[
\text{Volume} = \frac{6}{5} \times \frac{4}{5} \text{ cubic centimeters}
\]

So, the volume of Jorge’s rectangular prism is \(24\) cubic centimeters.
Lesson 11.10

Algebra • Apply Volume Formulas

You can use a formula to find the volume of a rectangular prism.

\[
Volume = length \times width \times height \\
V = (l \times w) \times h
\]

Find the volume of the rectangular prism.

Step 1 Identify the length, width, and height of the rectangular prism.

length = \(9\) in. \ width = \(3\) in. \ height = \(4\) in.

Step 2 Substitute the values of the length, width, and height into the formula.

\[
V = (l \times w) \times h \\
V = (9 \times 3) \times 4
\]

Step 3 Multiply the length by the width.

\[
V = (9 \times 3) \times 4 \\
V = 27 \times 4
\]

Step 4 Multiply the product of the length and width by the height.

\[
V = 27 \times 4 \\
V = 108
\]

So, the volume of the rectangular prism is \(108\) cubic inches.

More information on this strategy is available on Animated Math Model #44.
Lesson 11.11

Problem Solving • Compare Volumes

A company makes aquariums that come in three sizes of rectangular prisms. The length of each aquarium is three times its width and depth. The depths of the aquariums are 1 foot, 2 feet, and 3 feet. What is the volume of each aquarium?

<table>
<thead>
<tr>
<th>Read the Problem</th>
<th>Solve the Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What do I need to find?</strong></td>
<td><strong>Think:</strong> The depth of an aquarium is the same as the height of the prism formed by the aquarium.</td>
</tr>
<tr>
<td>I need to find the volume of each aquarium.</td>
<td></td>
</tr>
<tr>
<td><strong>What information do I need to use?</strong></td>
<td></td>
</tr>
<tr>
<td>I can use the formula for volume, $V = l \times w \times h$, or $V = B \times h$. I can use 1 ft, 2 ft, and 3 ft as the depths. I can use the clues the length is three times the width and depth.</td>
<td></td>
</tr>
<tr>
<td><strong>How will I use the information?</strong></td>
<td>So, the volumes of the aquariums are 3 cubic feet, 24 cubic feet, and 81 cubic feet.</td>
</tr>
<tr>
<td>I will use the volume formula and a table to list all of the possible combinations of lengths, widths, and depths.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length (ft)</th>
<th>Width (ft)</th>
<th>Depth, or Height (ft)</th>
<th>Volume (cu ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>81</td>
</tr>
</tbody>
</table>

More information on this strategy is available on Animated Math Model #44.
Lesson 11.12

Find Volume of Composed Figures

A composite figure is a solid made up of two or more solids. To find the volume of a composite figure, first find the volume of each solid that makes up the figure. Then find the sum of the volumes of the figures.

Find the volume of the composite figure at right.

**Step 1** Break apart the composite figure into two rectangular prisms. Label the dimensions of each prism.

<table>
<thead>
<tr>
<th>Prism 1</th>
<th>Prism 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 in.</td>
<td>4 in.</td>
</tr>
<tr>
<td>4 in.</td>
<td>4 in.</td>
</tr>
<tr>
<td>20 in.</td>
<td>20 in.</td>
</tr>
<tr>
<td>8 in.</td>
<td>6 in.</td>
</tr>
</tbody>
</table>

**Step 2** Find the volume of each prism.

- **Prism 1**
  - $V = (l \times w) \times h$
  - $V = \frac{4}{8} \times 4$
  - $V = 128 \text{ in.}^3$

- **Prism 2**
  - $V = (l \times w) \times h$
  - $V = \frac{20}{8} \times 4$
  - $V = 640 \text{ in.}^3$

**Step 3** Find the sum of the volumes of the two prisms.

- Volume of Prism 1 + Volume of Prism 2 = Volume of Composite Figure
  - $\frac{128 \text{ in.}^3}{768 \text{ in.}^3} = \frac{640 \text{ in.}^3}{768 \text{ in.}^3}$

So, the volume of the composite figure is 768 in.$^3$.

More information on this strategy is available on Animated Math Model #44.

**Vocabulary**

**Base** – a plane figure that is usually a polygon, used to describe and help find the volume of a solid figure

**Congruent** – having the same size and the same shape

**Lateral face** – a polygon that connects with the bases of a polyhedron

**Polygon** – a closed plane figure formed by three or more line segments that meet at points called vertices

**Polyhedron** – a solid figure with faces that are polygons
**Prism** – a solid figure with two congruent polygons that are bases, connected with lateral faces that are rectangles

**Regular polygon** -- a polygon in which all sides are congruent and all angles are congruent

**Unit cube** – a cube that has a length, width, and height of 1 unit

**Volume** – the measure of the amount of space a solid figure occupies