Example for each lesson:

Lesson 4.1

**Estimate Quotients Using Multiples**

Find two numbers the quotient of $142 \div 5$ is between. Then estimate the quotient.

You can use multiples to estimate. A *multiple* of a number is the product of a number and a counting number.

**Step 1** Think: What number multiplied by 5 is about 142?

Since 142 is greater than $10 \times 5$, or 50, use counting numbers 10, 20, 30, and so on to find multiples of 5.

**Step 2** Multiply 5 by multiples of 10 and make a table.

<table>
<thead>
<tr>
<th>Counting Number</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple of 5</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
</table>

**Step 3** Use the table to find multiples of 5 closest to 142.

$20 \times 5 = \frac{100}{5}$

$30 \times 5 = \frac{150}{5}$

142 is between $\frac{100}{5}$ and $\frac{150}{5}$.

142 is closest to $\frac{150}{5}$, so $142 \div 5$ is about $\frac{30}{5}$. 
Lesson 4.2

Remainders

Use counters to find the quotient and remainder.

\[
9 \div 26
\]

• Use 26 counters to represent the dividend, 26.
• Since you are dividing 26 by 9, draw 9 circles. Divide the 26 counters into 9 equal-sized groups.

• There are 2 counters in each circle, so the quotient is 2. There are 8 counters left over, so the remainder is 8.

\[
\begin{array}{c}
9 \longdiv{26} \\
2 \\
\hline
8
\end{array}
\]

Divide. Draw a quick picture to help.

\[
7 \div 66
\]

• Use 66 counters to represent the dividend, 66.
• Since you are dividing 66 by 7, draw 7 circles. Divide 66 counters into 7 equal-sized groups.

• There are 9 counters in each circle, so the quotient is 9. There are 3 counters left over, so the remainder is 3.

\[
\begin{array}{c}
7 \longdiv{66} \\
9 \\
\hline
3
\end{array}
\]

More information on this strategy is available on Animated Math Model #13.
Lesson 4.3

Interpret the Remainder

When you solve a division problem with a remainder, the way you interpret the remainder depends on the situation and the question.

<table>
<thead>
<tr>
<th>Way 1: Write the remainder as a fraction.</th>
<th>Way 2: Drop the remainder.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Callie has a board that is 60 inches long. She wants to cut 8 shelves of equal length from the board and use the entire board. How long will each shelf be?</td>
<td>Callie has 60 beads. She wants to make 8 identical bracelets and use as many beads as possible on each bracelet. How many beads will be on each bracelet?</td>
</tr>
<tr>
<td>Divide. (60 \div 8) (7\ \text{r}4)</td>
<td>Divide. (60 \div 8) (7\ \text{r}4)</td>
</tr>
</tbody>
</table>

The remainder, 4 inches, can be divided into 8 equal parts.

The remainder is the number of beads left over. Those beads will not be used. Drop the remainder.

\[
\frac{4}{8} = \text{remainder} \quad \frac{7}{8} = \text{divisor}
\]

Write the remainder as a fraction.

Callie will use \(7\) beads on each bracelet.

Each shelf will be \(\frac{7}{8}\) inches long.

<table>
<thead>
<tr>
<th>Way 3: Add 1 to the quotient.</th>
<th>Way 4: Use only the remainder.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Callie has 60 beads. She wants to put 8 beads in each container. How many containers will she need?</td>
<td>Callie has 60 stickers. She wants to give an equal number of stickers to 8 friends. She will give the leftover stickers to her sister. How many stickers will Callie give to her sister?</td>
</tr>
<tr>
<td>Divide. (60 \div 8) (7\ \text{r}4)</td>
<td>Divide. (60 \div 8) (7\ \text{r}4)</td>
</tr>
</tbody>
</table>

The answer shows that Callie can fill 7 containers but will have 4 beads left over. She will need 1 more container for the 4 leftover beads. Add 1 to the quotient.

The remainder is the number of stickers left over. Use the remainder as the answer.

Callie will need \(8\) containers.

Callie will give her sister \(4\) stickers.
Lesson 4.4

Divide Tens, Hundreds, and Thousands

You can use base-ten blocks, place value, and basic facts to divide.

Divide. $240 \div 3$

<table>
<thead>
<tr>
<th>Use base-ten blocks.</th>
<th>Use place value.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong> Draw a quick picture to show 240.</td>
<td><strong>Step 1</strong> Identify the basic fact to use.</td>
</tr>
<tr>
<td></td>
<td>Use $24 \div 3$.</td>
</tr>
<tr>
<td><strong>Step 2</strong> You cannot divide 2 hundreds into 3 equal groups. Rename 2 hundreds as tens.</td>
<td><strong>Step 2</strong> Use place value to rewrite 240 as tens.</td>
</tr>
<tr>
<td>$240 = 24$ tens</td>
<td>$240 = 24$ tens</td>
</tr>
<tr>
<td><strong>Step 3</strong> Separate the tens into 3 equal groups to divide.</td>
<td><strong>Step 3</strong> Divide.</td>
</tr>
<tr>
<td>There are 3 groups of $8$ tens. Write the answer.</td>
<td>$24$ tens $\div 3 = 8$ tens</td>
</tr>
<tr>
<td>$240 \div 3 = 80$</td>
<td>$= 80$</td>
</tr>
</tbody>
</table>

More information on this strategy is available on Animated Math Model #14.

Lesson 4.5

Estimate Quotients Using Compatible Numbers

Compatible numbers are numbers that are easy to compute mentally. In division, one compatible number divides evenly into the other. Think of the multiples of a number to help you find compatible numbers.

Estimate. $6 \div 216$

**Step 1** Think of these multiples of 6:

<table>
<thead>
<tr>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
<th>36</th>
<th>42</th>
<th>48</th>
<th>54</th>
</tr>
</thead>
</table>

Find multiples that are close to the first 2 digits of the dividend. $18$ tens and $24$ tens are both close to $21$ tens. You can use either or both numbers to estimate the quotient.

**Step 2** Estimate using compatible numbers.

$216 \div 6$

$216 \div 6$

$180 \div 6 = 30$

$240 \div 6 = 40$

So, $216 \div 6$ is between $30$ and $40$.

**Step 3** Decide whether the estimate is closer to 30 or 40.

$216 - 180 = 36$

$240 - 216 = 24$

$216$ is closer to 240, so use $40$ as the estimate.

More information on this strategy is available on Animated Math Model #15.
Lesson 4.6

**Division and the Distributive Property**

Divide. $78 \div 6$

Use the Distributive Property and quick pictures to break apart numbers to make them easier to divide.

**Step 1** Draw a quick picture to show 78.

**Step 2** Think about how to break apart 78. You know 6 tens $\div 6 = 10$, so use $78 = 60 + 18$. Draw a quick picture to show 6 tens and 18 ones.

**Step 3** Draw circles to show 6 tens $\div 6$ and 18 ones $\div 6$. Your drawing shows the use of the Distributive Property.

$78 \div 6 = (60 \div 6) + (18 \div 6)$

**Step 4** Add the quotients to find $78 \div 6$.

$78 \div 6 = (60 \div 6) + (18 \div 6)$

$= 10 + 3$

$= 13$

Lesson 4.7

**Divide Using Repeated Subtraction**

You can use repeated subtraction to divide. Use repeated subtraction to solve the problem.

Nestor has 27 shells to make bracelets. He needs 4 shells for each bracelet. How many bracelets can he make?

**Divide. 27 $\div 4$**

Write $4)27$.

**Step 1** Subtract the divisor until the remainder is less than the divisor.

Subtract the divisor

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Record a 1 each time you subtract.

So, Nestor can make 6 bracelets. He will have 3 shells left.

**Step 2** Count the number of times you subtracted the divisor, 4.

4 is subtracted six times with 3 left.

$27 \div 4$

$= 6 \text{ r} 3$

More information on this strategy is available on Animated Math Model #16.
Divide Using Partial Quotients

You can use partial quotients to divide.

**Divide.** $492 \div 4$

**Step 1** Subtract greater multiples of the divisor. Repeat if needed.

\[
\begin{array}{c}
4)492 \\
\underline{- 400} \\
92 \\
\underline{- 80} \\
12 \\
\underline{- 12} \\
0
\end{array}
\] 100 $\times$ 4 $\rightarrow$ 100

**Step 2** Subtract lesser multiples of the divisor. Repeat until the remaining number is less than the divisor.

\[
\begin{array}{c}
4)492 \\
\underline{- 400} \\
92 \\
\underline{- 80} \\
12 \\
\underline{- 12} \\
0
\end{array}
\] 20 $\times$ 4 $\rightarrow$ 20

\[
\begin{array}{c}
4)492 \\
\underline{- 400} \\
92 \\
\underline{- 80} \\
12 \\
\underline{- 12} \\
0
\end{array}
\] 3 $\times$ 4 $\rightarrow$ 3

**Step 3** Add the partial quotients.

\[
\begin{array}{c}
100 \\
20 \\
3
\end{array}
\] $\rightarrow$ 123

Use rectangular models to record partial quotients.

\[
\begin{array}{c}
100 \\
400 \\
400 \\
400
\end{array}
\] $\rightarrow$ 492

\[
\begin{array}{c}
80 \\
12 \\
80 \\
80
\end{array}
\] $\rightarrow$ 400

\[
\begin{array}{c}
20 \\
12 \\
12 \\
12
\end{array}
\] $\rightarrow$ 92

\[
\begin{array}{c}
3
\end{array}
\] $\rightarrow$ 123
Lesson 4.9

Model Division with Regrouping

You can use base-ten blocks to model division with regrouping.

**Use base-ten blocks to find the quotient**  \(65 \div 4\).

**Step 1** Show 65 with base-ten blocks.

**Step 2** Draw 4 circles to represent dividing 65 into 4 equal groups. Share the tens equally among the 4 groups.

**Step 3** Regroup leftover tens as ones.

**Step 4** Share the ones equally among the 4 groups.

There are 1 ten(s) and 6 one(s) in each group with 1 left over.

So, the quotient is 16 r1.
Lesson 4.10

Place the First Digit

Divide. $763 \div 3 = \underline{\hspace{2cm}}$

**Step 1** Estimate. Then divide the hundreds.

**Think:** $3 \times 1$ hundred = 3 hundreds
$3 \times 2$ hundreds = 6 hundreds
$3 \times 3$ hundreds = 9 hundreds

$3 \times 3$ hundreds is too large.
Use 2 hundreds as an estimate.

2 \hline
3)763
-6
\hline
16
Bring down the 6.

**Step 2** Bring down the tens digit. Then divide the tens.

25 \hline
3)763
-6
\hline
16
Bring down the 6.

**Step 3** Bring down the ones digit. Then divide the ones.

254 \hline
3)763
-6
\hline
16
Bring down the 3.

254 \hline
3)763
-6
\hline
16
-15
\hline
13
Multiply, $3 \times 4$ ones
\hline
1
Subtract.

**Step 4** Check to make sure that the remainder is less than the divisor. Write the answer.

$254 \text{ r} 1$ \hline
3)763
-6
\hline
16
-15
\hline
13
-12
\hline
1
Subtract.

More information on this strategy is available on Animated Math Model #17.
### Divide by 1-Digit Numbers

**Divide.** \(766 \div 6 = \) 

1. **Step 1** Use place value to place the first digit. **Think:** 7 hundreds can be shared among 6 groups without regrouping. 

   \[
   \begin{array}{c|c}
   1 & 6)766 \\
   6 & \multicolumn{1}{c}{\text{---}} \\
   \hline
   6 & 6 \\
   \hline
   16 & \text{Bring down the 6.}
   \end{array}
   \]

2. **Step 2** Bring down the tens digit. Then divide the tens. 

   \[
   \begin{array}{c|c}
   12 & 6)766 \\
   6 & \multicolumn{1}{c}{\text{---}} \\
   \hline
   6 & 6 \\
   \hline
   16 & \text{Divide 16 tens by 6.}
   \end{array}
   \]

3. **Step 3** Bring down the ones digit. Then divide the ones. 

   \[
   \begin{array}{c|c}
   127 & 6)766 \\
   6 & \multicolumn{1}{c}{\text{---}} \\
   \hline
   6 & 6 \\
   \hline
   16 & \text{Multiply, } 6 \times 2 \text{ tens} \\
   \hline
   12 & \text{Subtract.}
   \end{array}
   \]

4. **Step 4** Check to make sure that the remainder is less than the divisor. Write the answer. 

   \[
   \begin{array}{c|c}
   127 \text{ r}4 & 6)766 \\
   6 & \multicolumn{1}{c}{\text{---}} \\
   \hline
   12 & 4 \text{ < 6}
   \end{array}
   \]

5. **Step 5** Use multiplication and addition to check your answer. 

   \[
   \begin{align*}
   127 \times 6 & = 762 \\
   + 4 & = 766
   \end{align*}
   \]

More information on this strategy is available on Animated Math Models #18, 19.
Lesson 4.12

Problem Solving • Multistep Division Problems

There are 72 third graders and 84 fourth graders going on a field trip. An equal number of students will ride on each of 4 buses. How many students will ride on each bus?

<table>
<thead>
<tr>
<th>Read the Problem</th>
<th>Solve the Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What do I need to find?</strong></td>
<td>I can model the number of students in all using a bar diagram.</td>
</tr>
<tr>
<td>I need to find the number of students who will ride on each bus.</td>
<td>72 84</td>
</tr>
<tr>
<td></td>
<td>156</td>
</tr>
<tr>
<td><strong>What information do I need to use?</strong></td>
<td>I can model the number of buses and divide to find the number of students on each bus.</td>
</tr>
<tr>
<td>There are 72 third graders and 84 fourth graders. There will be 4 buses.</td>
<td>39 39 39 39</td>
</tr>
<tr>
<td></td>
<td>156</td>
</tr>
<tr>
<td><strong>How will I use the information?</strong></td>
<td>So, 39 students will ride on each bus.</td>
</tr>
<tr>
<td>I will make a bar diagram for each step. I will add 72 and 84 to find the total number of students. I will divide by 4 to find how many students will ride on each bus.</td>
<td></td>
</tr>
</tbody>
</table>

Vocabulary

Compatible numbers – numbers that are easy to compute with mentally

Multiple – a number that is the product of a given number and a counting number

Partial quotient – a method of dividing in which multiples of the divisor are subtracted from the dividend and then the quotients are added together

Remainder – the amount left over when a number cannot be divided equally

Dividend – the number that is to be divided in a division problem

Divisor – the number that divides the dividend